HP 12C Sin, Sin^{-1}, Cos, Cos^{-1}, Tan, Tan^{-1}, Quartic, Parallel Resistances, Geometric Mean, Solver

PROGRAM			STACK Registers					Comments
Label	No	Instruction	X	Y	Z	T	L	20220824 J E Patterson jepspectro.com
	1	GTO 65						Start Solver
	2	R/S	Return from function and stop					GTO 08 GTO 07 or GTO 04 for $trig^{-1}$
	3	GTO 11						Bhaskara I GTO 11, Quartic GTO 34
	4	X↔Y						
	5	÷						tan(θ). Solver returns θ from $tan^{-1}(x)$
	6	GTO 08						
	7	X↔Y						cos(θ). Solver returns θ from $cos^{-1}(x)$
	8	RCL4						sin(θ). Solver returns θ from $sin^{-1}(x)$
	9	-						
	10	GTO 70						Return to solver
	11	6						Bhaskara I equation for cosine
	12	0						
	13	÷						
	14	ENTER						
	15	x						
	16	9						
	17	X↔Y						
	18	+						
	19	9						
	20	LST X						
	21	4						
	22	x						
	23	-						
	24	X↔Y						
	25	÷						Cosine result
	26	ENTER						Calculate sine
	27	ENTER						$\sin(x) = \sqrt{(1-\cos(x)^2)}$
	28	x						
	29	1						
	30	X↔Y						
	31	-						
	32	√X						sin(x) displayed, X<>Y for cos(x)
	33	GTO 02						
	34	ENTER						Quartic equation
	35	ENTER						
	36	ENTER						
	37	RCL n	Quartic - enter zero for lower order					a4 store in finance register n
	38	x						
	39	RCL i	Cubic - enter zero for lower order					a3 store in finance register i
	40	+						
	41	x						
	42	RCL PV	Quadratic					a2 store in finance register PV
	43	+						
	44	x						
	45	RCL PMT	Linear					a1 store in finance register PMT
	46	+						
	47	x						
	48	RCL FV	Constant					a0 store in finance register FV
	49	+						
	50	GTO 70						Return to solver
	51	R/S						Parallel resistors
	52	1/X						R1 ENTER R2 R/S R3 R/S etc.
	53	X↔Y						
	54	1/X						
	55	+						
	56	1/X						
	57	GTO 51						Goto 51 and wait for next input R3
	58	Σ						Geometric mean - clear stats regs
	59	R/S						First input
	60	LN						
	61	Σ+						
	62	$\bar{x}$						

	63	e^x						
	64	GTO 59						Goto 59 and wait for input
■	65	STO 1						Newton Solver - initial guess
	66	1						Loop return point and set flag to 1
	67	STO 0						Flag = 1
	68	RCL 1						Initial guess
	69	GTO 03						Call f(x) and return to 70
	70	RCL 0						Function calls return here
	71	x=0?						Is call from f(x+h) where flag = 0
	72	GTO 87						Then continue with Newton correction
	73	STO-0						Else change flag to 0
	74	R↓						Roll down to f(x) result
	75	STO 2						Save f(x), tending to 0 as x changes
	76	RCL 1						Calculate h
	77	x=0?						Avoid possible divide by zero
	78	e^x						h = x/10000, or 1/10000 if x=0
	79	EEX						
	80	4						
	81	÷						h = x/10000, or 1/10000 if x=0
	82	STO 3						Store h, initially 1E-4
	83	RCL 1						x, adjust until f(x) = 0
	84	PSE						Show progress
	85	+						x+h
	86	GTO 03						Call f(x+h) then return to 70
	87	R↓						Roll down and display new f'(x)
	88	RCL 2						Old f(x) to calculate correction
	89	-						(f'(x)-f(x))
	90	RCL 3						h was checked for zero at line 77
	91	÷						(f'(x)-f(x))/h
	92	STO÷2						Adjust f(x) towards zero
	93	RCL 2						new f(x) correction term
	94	STO-1						Adjust x so f(x) is closer to 0
	95	RND						Round according to decimal places
	96	x=0?						Is f(x)=0?
	97	GTO 99						Then finish and display result
	98	GTO 66						Else loop
	99	RCL 1						Display result

The black rectangles on the left mark program segments. They are described briefly here.

Line 1 starts the Newton Solver.

Line 2 chooses $\text{Sin}^{-1}(\theta)$, $\text{Cos}^{-1}(\theta)$ and $\text{Tan}^{-1}(\theta)$ at **Lines 08, 07** or **04** to be calculated by the Newton Solver. There is also the **R/S** option to not use the Newton Solver. In this case Sin(θ), Cos(θ) and Tan(θ) can be directly accessed from the Bhaskara I equation. Sin(θ), in stack level x, is displayed, Cos(θ) is in stack level y.

Line 3 chooses either the trig functions at **Line 11** or the solution to a Quartic polynomial at **Line 34**, using the Newton Solver.

Lines 04 to **10** selects the correct inverse trig function to solve for θ.

Line 11 is the Bhaskara I cosine equation which estimates Sin(θ), press **X<>Y** to get Cos(θ) and press ÷ to get Tan(θ).

Line 34 is a Quartic equation $a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$. It is programmed in the form $(((a_4x + a_3)x + a_2)x + a_1)x + a_0 = 0$. The constants a_4 to a_0 are entered into the financial registers **n**, **i**, **PV**, **PMT** and **FV**. These can be used as normal registers. Zero is entered for each missing equation term. A guess for x is entered and **GTO 00 R/S** starts the Newton Solver. To solve $x^4 - 5x^2 + 4 = 0$, enter 1 into **n**, 0 into **i**, -5 into **PV**, 0 into **PMT** and 4 into **FV**. Suitable guesses are -3, -1.3, 1.3 and 3. The four roots are -2. -1, 1 and 2.

Line 52 calculates parallel resistances. **GTO 52** R1 **ENTER** R2 **R/S** to start. To parallel an additional resistance enter R3 **R/S**. For a harmonic mean from 1 to 6: 1 **ENTER** 2 **R/S**, 3 **R/S**, 4 **R/S**, 5 **R/S**, 6 **R/S** and multiply by the number of entries.

Line 58 calculates a geometric mean. **GTO 58 R/S**, clears the statistics registers. x1 **R/S** x2 **R/S** x3 **R/S** etc. The geometric mean is progressively displayed.

Line 65 is the solver start point. Above this line any other equation of interest could be substituted, but preferably below **Line 3**. Functions return to **Line 70**. The solver loops back to **Line 66** if f(x) is not equal to 0, else **Line 99**.

A brief reminder card could be added to the calculator slip-case.